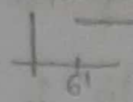
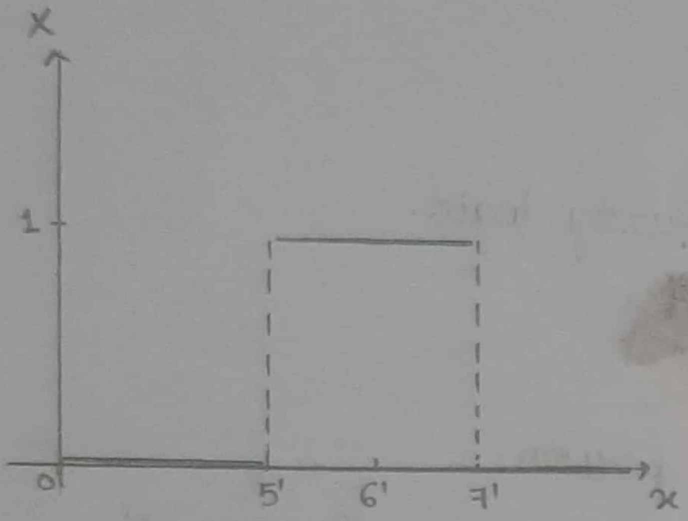
CLASSICAL SETS

~~membership funct~~

$$\rightarrow X_H = \begin{cases} 1 & ; \quad x \geq 6' \\ 0 & ; \quad x < 6' \end{cases}$$



→ Range of ht 5'-7'



• Open loop

FUZZY SETS

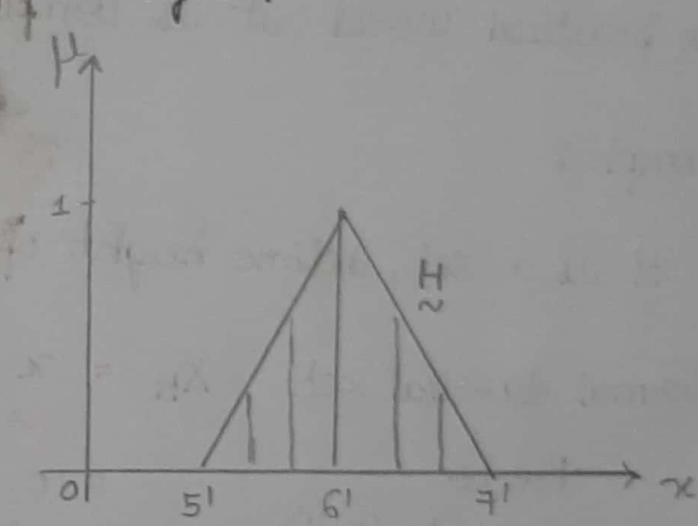
→ Around 6'

membership funct → μ

$\mu_{\underset{\sim}{H}}$ or $\mu_{\tilde{H}}$, $\mu_{\underset{\cdot}{H}}$, $\mu_{\dot{H}}$

$\mu_{\underset{\sim}{H}}(x) \in [0, 1]$

↳ ranges from 0 to 1



Having height around 6'

• closed loop having various degree of representation

only 6' → 1

other height → diff values

Complete set of element called Universal X if it is continous & infinite then fuzzy set is

$$\underset{\sim}{A} = \int \frac{\mu_{\underset{\sim}{A}}(x_i)}{x}$$

If it is finite & discrete $\underset{\sim}{A} = \sum \frac{\mu_{\underset{\sim}{A}}(x_i)}{x_i}$

Example :

$$\underset{\sim}{A} = \left\{ \frac{0.1}{1} + \frac{0.5}{2} + \frac{0.3}{3} + \frac{0.4}{4} + \frac{0.9}{5} \right\}$$

elements → 5

membership value of element 1 is 0.1.

$$\underset{\sim}{A} = \left\{ \frac{0.1}{1} + \quad + \frac{0.3}{3} + \frac{0.7}{4} \right\}$$

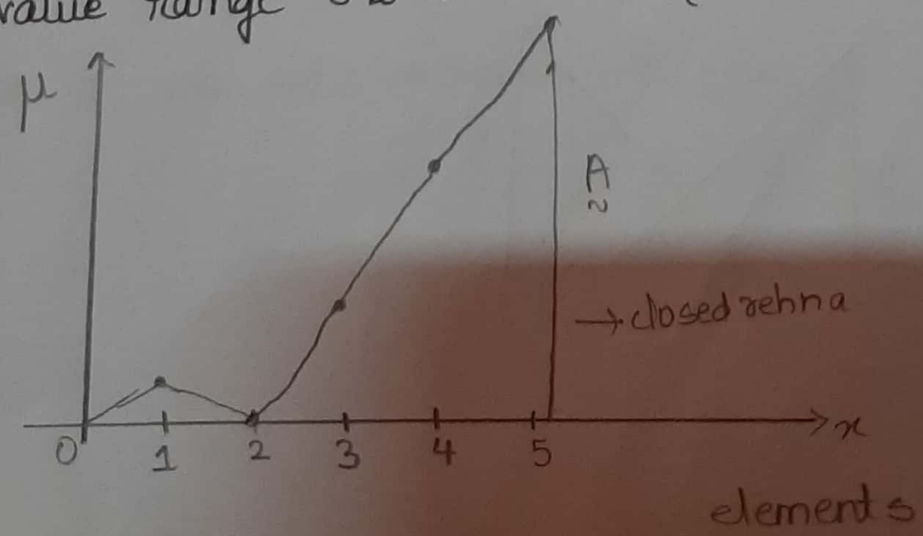
element 2 → membership = 0

FUZZY SET :

→ Mapping the element in the universe whose membership is in the range 0 to 1

- Sets containing element, each element is mapped to membership whose value range 0 to 1

$$\underset{\sim}{A} = \left\{ \frac{0.1}{1} + \quad \frac{0.3}{3} + \frac{0.7}{4} + \frac{0.9}{5} \right\}$$



OPERATIONS ON FUZZY SETS :

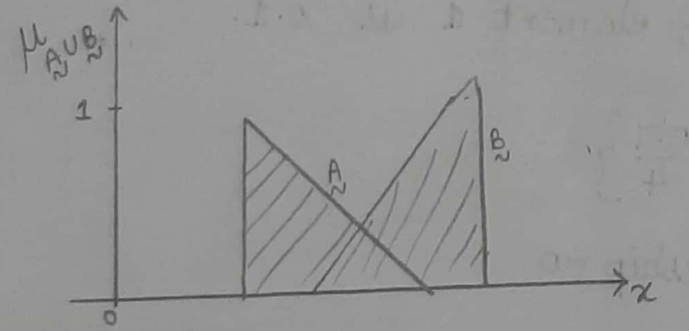
- union operation
- intersection operation
- compliment operation

→ Let us consider three fuzzy sets $\underset{\sim}{A}$, $\underset{\sim}{B}$ & $\underset{\sim}{C}$

UNION :

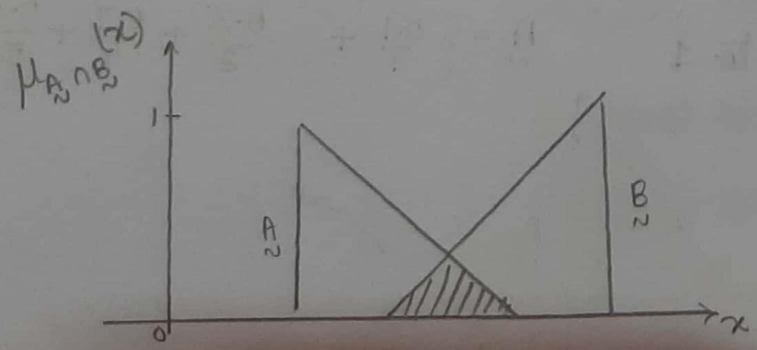
$$\mu_{\underset{\sim}{A} \cup \underset{\sim}{B}}(x) = \mu_{\underset{\sim}{A}}(x) \vee \mu_{\underset{\sim}{B}}(x)$$

membership value range 0-1



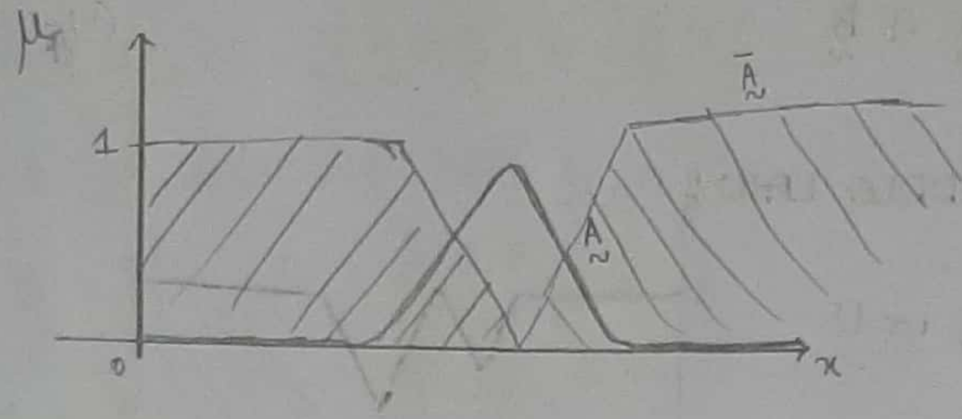
INTERSECTION :

$$\mu_{\underset{\sim}{A} \cap \underset{\sim}{B}}(x) = \mu_{\underset{\sim}{A}}(x) \wedge \mu_{\underset{\sim}{B}}(x)$$



COMPLEMENT :

$$\mu_{\bar{\underset{\sim}{A}}}(x) = 1 - \mu_{\underset{\sim}{A}}(x)$$



X → universal set   Property :.

1. A ⊆ X $\Rightarrow$ $\mu_{\underset{\sim}{A}}(x) \leq \mu_X(x)$

(A is a subset of X)

2. ϕ → null set

all elements will have membership of zero

~~whole set~~ whole set → X

all elements will have membership of 1.

2. For all $x \in X$, $\mu_{\phi}(x) = 0$

3. For all $x \in X$, $\mu_X(x) = 1$ (X → whole set).

Collection of all sets → power set $\underset{\sim}{P}(X)$

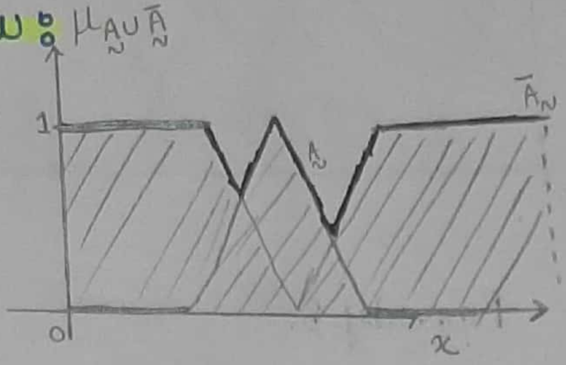
DE MORGAN'S LAW : ← 2m + proof.

$$\overline{\underset{\sim}{A} \cap \underset{\sim}{B}} = \overline{\underset{\sim}{A}} \cup \overline{\underset{\sim}{B}}$$

$$\overline{\underset{\sim}{A} \cup \underset{\sim}{B}} = \overline{\underset{\sim}{A}} \cap \overline{\underset{\sim}{B}}$$

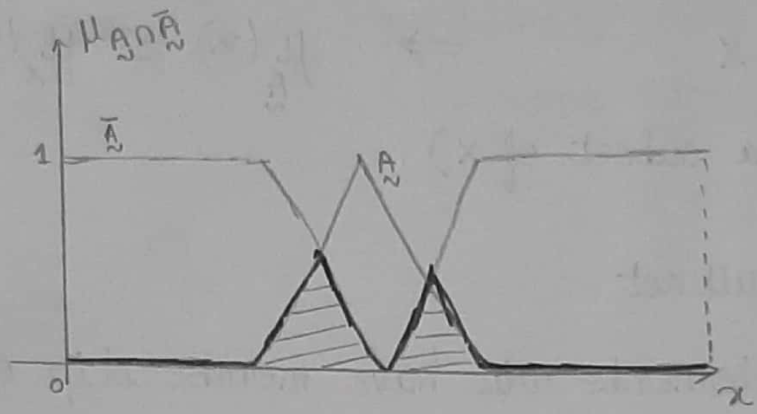
EXCLUDED MIDDLE LAW :
(for fuzzy set).

$$\underset{\sim}{A} \cup \overline{\underset{\sim}{A}} \neq X \text{ or } U$$



$$\underset{\sim}{A} \cap \overline{\underset{\sim}{A}} \neq \phi$$

(law of contradiction)



PROPERTIES OF FUZZY SET :

[10m: operation + proof properties + sum]

1. COMMUTATIVE : $\underset{\sim}{A} \cup \underset{\sim}{B} = \underset{\sim}{B} \cup \underset{\sim}{A}$

2. ASSOCIATIVE :

$$\underset{\sim}{A} \cup (\underset{\sim}{B} \cup \underset{\sim}{C}) = (\underset{\sim}{A} \cup \underset{\sim}{B}) \cup \underset{\sim}{C}$$

$$\underset{\sim}{A} \cap (\underset{\sim}{B} \cap \underset{\sim}{C}) = (\underset{\sim}{A} \cap \underset{\sim}{B}) \cap \underset{\sim}{C}$$

3. DISTRIBUTIVE :

$$\underset{\sim}{A} \cup (\underset{\sim}{B} \cap \underset{\sim}{C}) = (\underset{\sim}{A} \cup \underset{\sim}{B}) \cap (\underset{\sim}{A} \cup \underset{\sim}{C})$$

$$\underset{\sim}{A} \cap (\underset{\sim}{B} \cup \underset{\sim}{C}) = (\underset{\sim}{A} \cap \underset{\sim}{B}) \cup (\underset{\sim}{A} \cap \underset{\sim}{C})$$

4. IDENTITY : ~~or law of zero~~ :

$$\underset{\sim}{A} \cup \phi = \underset{\sim}{A} \quad \& \quad \underset{\sim}{A} \cap X = \underset{\sim}{A}$$

$$A \cap \phi = \phi \quad \& \quad \underset{\sim}{A} \cup X = X$$ (law of zero)

5. IDEMPOTENCY :

$$\underset{\sim}{A} \cup \underset{\sim}{A} = \underset{\sim}{A}$$

& $$\underset{\sim}{A} \cap \underset{\sim}{A} = \underset{\sim}{A}$$

6. TRANSITIVITY :

If $\underset{\sim}{A} \subseteq \underset{\sim}{B} \subseteq \underset{\sim}{C}$ then $\underset{\sim}{A} \subseteq \underset{\sim}{C}$

7. INVOLUTION : (Double Negation Law)

$$\overline{\overline{\underset{\sim}{A}}} = \underset{\sim}{A}$$

PROBLEM :

Let $\underset{\sim}{A} = \left\{ \frac{1}{2} + \frac{0.5}{3} + \frac{0.3}{4} + \frac{0.2}{5} \right\}$

$\underset{\sim}{B} = \left\{ \frac{0.5}{2} + \frac{0.7}{3} + \frac{0.2}{4} + \frac{0.4}{5} \right\}$

Calculate the compliment, union, intersection, difference & Prove the Demorgan's law.

Sol:-

$$\underset{\sim}{A} = \left\{ \frac{1}{2} + \frac{0.5}{3} + \frac{0.3}{4} + \frac{0.2}{5} \right\}$$

$$\underset{\sim}{A} = \left\{ \frac{0}{1} + \frac{1}{2} + \frac{0.5}{3} + \frac{0.3}{4} + \frac{0.2}{5} \right\}$$

$$\overline{\underset{\sim}{A}} = \left\{ \frac{1}{1} + \frac{0}{2} + \overset{1-0.5}{\frac{0.5}{3}} + \overset{1-0.3}{\frac{0.7}{4}} + \overset{1-0.2}{\frac{0.8}{5}} \right\}$$

$$\underset{\sim}{B} = \left\{ \frac{0}{1} + \frac{0.5}{2} + \frac{0.7}{3} + \frac{0.2}{4} + \frac{0.4}{5} \right\}$$

$$\overline{\underset{\sim}{B}} = \left\{ \frac{1}{1} + \frac{0.5}{2} + \frac{0.3}{3} + \frac{0.8}{4} + \frac{0.6}{5} \right\}$$

$$\underset{\sim}{A} \cup \underset{\sim}{B} = \left\{ \frac{0}{1} + \overset{\underbrace{1, 0.5}_{max}}{\frac{1}{2}} + \frac{0.7}{3} + \frac{0.3}{4} + \frac{0.4}{5} \right\}$$

$$\underset{\sim}{A} \cap \underset{\sim}{B} = \left\{ \frac{0}{1} + \overset{1, 0.5}{\frac{0.5}{2}} + \frac{0.5}{3} + \frac{0.2}{4} + \frac{0.2}{5} \right\}$$

difference ; $\underset{\sim}{A} | \underset{\sim}{B} = \underset{\sim}{A} \cap \overline{\underset{\sim}{B}} = \left\{ \frac{0}{1} + \frac{0.5}{2} + \frac{0.3}{3} + \frac{0.3}{4} + \frac{0.2}{5} \right\}$

$\underset{\sim}{B} | \underset{\sim}{A} = \underset{\sim}{B} \cap \overline{\underset{\sim}{A}} = \left\{ \frac{0}{1} + \frac{0}{2} + \frac{0.5}{3} + \frac{0.2}{4} + \frac{0.4}{5} \right\}$

$$\overline{\underset{\sim}{A} \cap \underset{\sim}{B}} = \left\{ \frac{1}{1} + \frac{0.5}{2} + \frac{0.5}{3} + \frac{0.8}{4} + \frac{0.8}{5} \right\}$$

$$\overline{\underset{\sim}{A}} \cup \overline{\underset{\sim}{B}} = \left\{ \frac{1}{1} + \frac{0.5}{2} + \frac{0.5}{3} + \frac{0.8}{4} + \frac{0.8}{5} \right\}$$

$$\overline{\underset{\sim}{A} \cap \underset{\sim}{B}} = \overline{\underset{\sim}{A}} \cup \overline{\underset{\sim}{B}}$$

$$\overline{\underset{\sim}{A} \cup \underset{\sim}{B}} = \left\{ \frac{1}{1} + \frac{0}{2} + \frac{0.3}{3} + \frac{0.7}{4} + \frac{0.6}{5} \right\}$$

$$\overline{\underset{\sim}{A}} \cap \overline{\underset{\sim}{B}} = \left\{ \frac{1}{1} + \frac{0}{2} + \frac{0.3}{3} + \frac{0.7}{4} + \frac{0.6}{5} \right\}$$

$$\overline{\underset{\sim}{A} \cup \underset{\sim}{B}} = \overline{\underset{\sim}{A}} \cap \overline{\underset{\sim}{B}}$$

Let $\underset{\sim}{C} = \left\{ \frac{0.2}{1} + \frac{0.4}{2} + \frac{0.5}{3} + \frac{0.3}{4} + \frac{0.7}{5} \right\}$ Prove

$$\underset{\sim}{A} = \left\{ \frac{0}{1} + \frac{1}{2} + \frac{0.5}{3} + \frac{0.3}{4} + \frac{0.2}{5} \right\}$$

$$\underset{\sim}{B} = \left\{ \frac{0}{1} + \frac{0.5}{2} + \frac{0.7}{3} + \frac{0.2}{4} + \frac{0.4}{5} \right\}$$

(i) Associative :

$$\underset{\sim}{A} \cup (\underset{\sim}{B} \cup \underset{\sim}{C}) = (\underset{\sim}{A} \cup \underset{\sim}{B}) \cup \underset{\sim}{C}$$

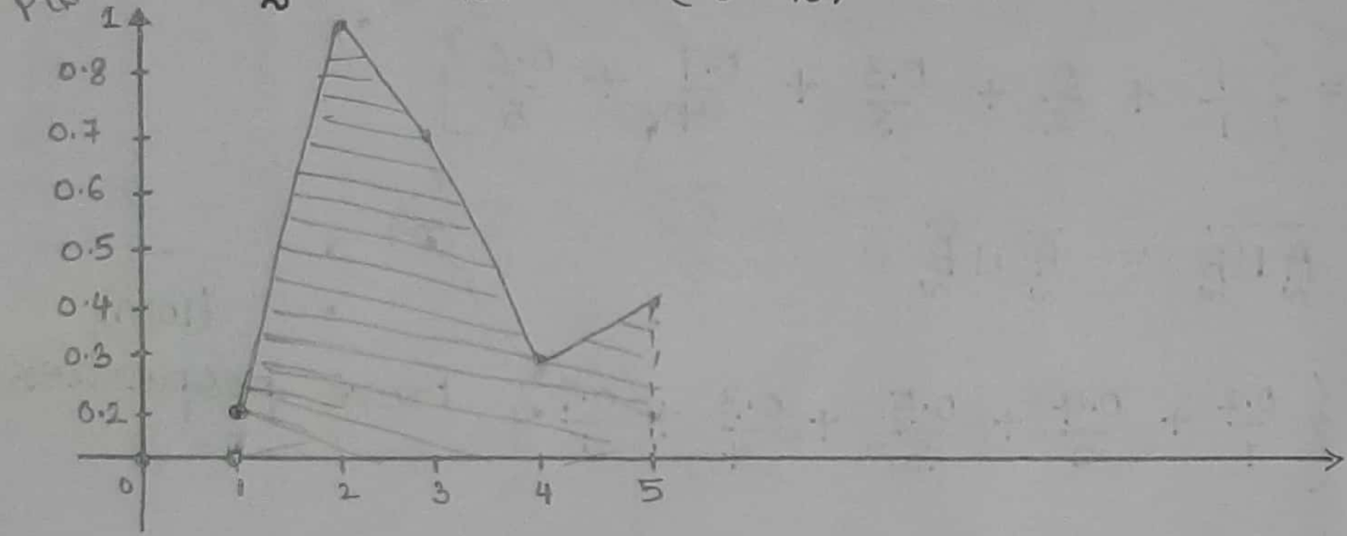
$$(\underset{\sim}{B} \cup \underset{\sim}{C}) = \left\{ \frac{0.2}{1} + \frac{0.5}{2} + \frac{0.7}{3} + \frac{0.3}{4} + \frac{0.7}{5} \right\}$$

$$\underset{\sim}{A} \cup (\underset{\sim}{B} \cup \underset{\sim}{C}) = \left\{ \frac{0.2}{1} + \frac{1}{2} + \frac{0.7}{3} + \frac{0.3}{4} + \frac{0.7}{5} \right\}$$

$$\underset{\sim}{A} \cup \underset{\sim}{B} = \left\{ \frac{0}{1} + \frac{1}{2} + \frac{0.7}{3} + \frac{0.3}{4} + \frac{0.4}{5} \right\}$$

$$(\underset{\sim}{A} \cup \underset{\sim}{B}) \cup \underset{\sim}{C} = \left\{ \frac{0.2}{1} + \frac{1}{2} + \frac{0.7}{3} + \frac{0.3}{4} + \frac{0.7}{5} \right\}$$

$$\underset{\sim}{A} \cup (\underset{\sim}{B} \cup \underset{\sim}{C}) = (\underset{\sim}{A} \cup \underset{\sim}{B}) \cup \underset{\sim}{C}$$



$$\underset{\sim}{A} \cap (\underset{\sim}{B} \cap \underset{\sim}{C}) = (\underset{\sim}{A} \cap \underset{\sim}{B}) \cap \underset{\sim}{C}$$

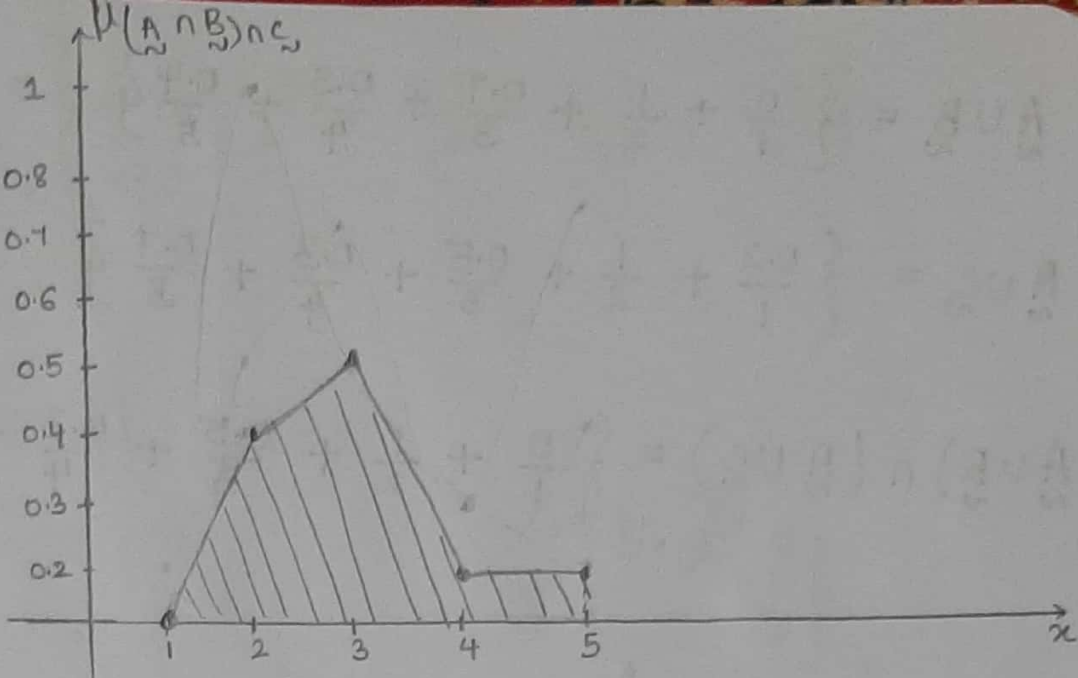
$$\underset{\sim}{B} \cap \underset{\sim}{C} = \left\{ \frac{0}{1} + \frac{0.4}{2} + \frac{0.5}{3} + \frac{0.2}{4} + \frac{0.4}{5} \right\}$$

$$\underset{\sim}{A} \cap (\underset{\sim}{B} \cap \underset{\sim}{C}) = \left\{ \frac{0}{1} + \frac{0.4}{2} + \frac{0.5}{3} + \frac{0.2}{4} + \frac{0.2}{5} \right\}$$

$$\underset{\sim}{A} \cap \underset{\sim}{B} = \left\{ \frac{0}{1} + \frac{0.5}{2} + \frac{0.5}{3} + \frac{0.2}{4} + \frac{0.2}{5} \right\}$$

$$(\underset{\sim}{A} \cap \underset{\sim}{B}) \cap \underset{\sim}{C} = \left\{ \frac{0}{1} + \frac{0.4}{2} + \frac{0.5}{3} + \frac{0.2}{4} + \frac{0.2}{5} \right\}$$

Scanned with CamScanner

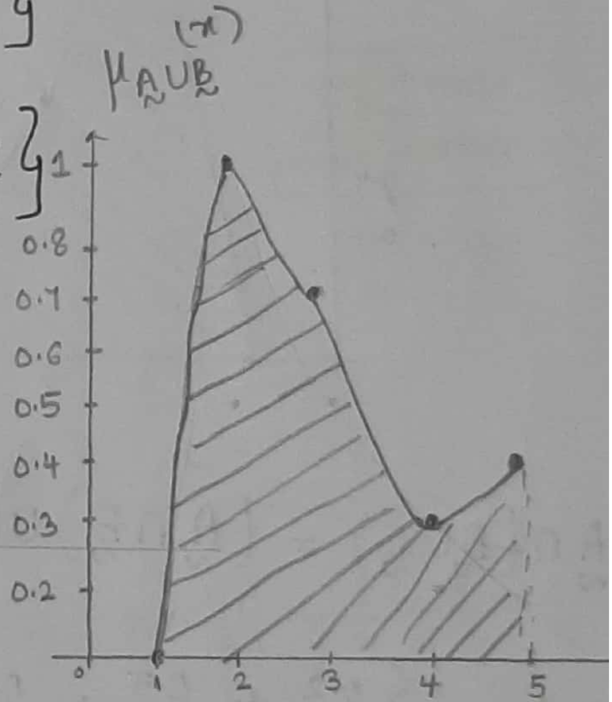


(ii) Commutative

$$\underset{\sim}{A} \cup \underset{\sim}{B} = \underset{\sim}{B} \cup \underset{\sim}{A}$$

$$\underset{\sim}{A} \cup \underset{\sim}{B} = \left\{ \frac{0}{1} + \frac{1}{2} + \frac{0.7}{3} + \frac{0.3}{4} + \frac{0.4}{5} \right\}$$

$$\underset{\sim}{B} \cup \underset{\sim}{A} = \left\{ \frac{0}{1} + \frac{1}{2} + \frac{0.7}{3} + \frac{0.3}{4} + \frac{0.4}{5} \right\}$$



(iii) Distributive

$$\underset{\sim}{A} \cup (\underset{\sim}{B} \cap \underset{\sim}{C}) = (\underset{\sim}{A} \cup \underset{\sim}{B}) \cap (\underset{\sim}{A} \cup \underset{\sim}{C})$$

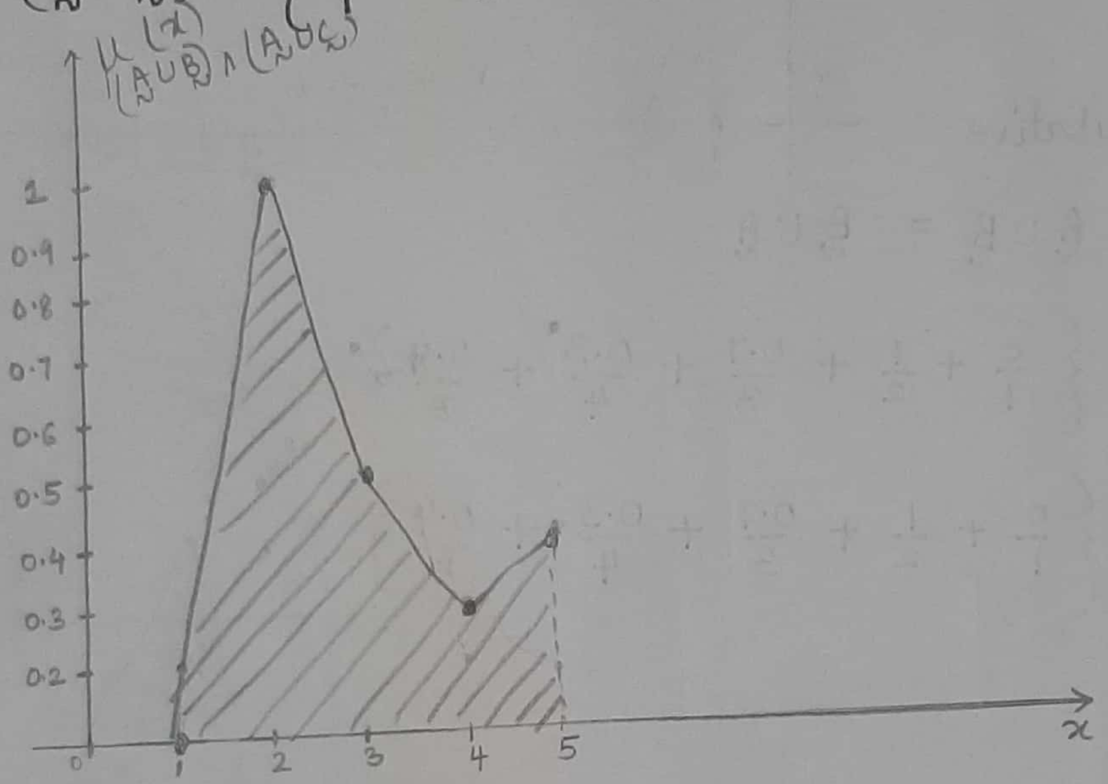
$$\underset{\sim}{B} \cap \underset{\sim}{C} = \left\{ \frac{0}{1} + \frac{0.4}{2} + \frac{0.5}{3} + \frac{0.2}{4} + \frac{0.4}{5} \right\}$$

$$\underset{\sim}{A} \cup (\underset{\sim}{B} \cap \underset{\sim}{C}) = \left\{ \frac{0}{1} + \frac{\cancel{0.5}\ 1}{2} + \frac{0.5}{3} + \frac{0.3}{4} + \frac{0.4}{5} \right\}$$

$$\underset{\sim}{A} \cup \underset{\sim}{B} = \left\{ \frac{0}{1} + \frac{1}{2} + \frac{0.7}{3} + \frac{0.3}{4} + \frac{0.4}{5} \right\}$$

$$\underset{\sim}{A} \cup \underset{\sim}{C} = \left\{ \frac{0.2}{1} + \frac{1}{2} + \frac{0.5}{3} + \frac{0.3}{4} + \frac{0.7}{5} \right\}$$

$$(\underset{\sim}{A} \cup \underset{\sim}{B}) \cap (\underset{\sim}{A} \cup \underset{\sim}{C}) = \left\{ \frac{0}{1} + \frac{1}{2} + \frac{0.5}{3} + \frac{0.3}{4} + \frac{0.4}{5} \right\}$$



$$\underset{\sim}{A} \cap (\underset{\sim}{B} \cup \underset{\sim}{C}) = (\underset{\sim}{A} \cap \underset{\sim}{B}) \cup (\underset{\sim}{A} \cap \underset{\sim}{C})$$

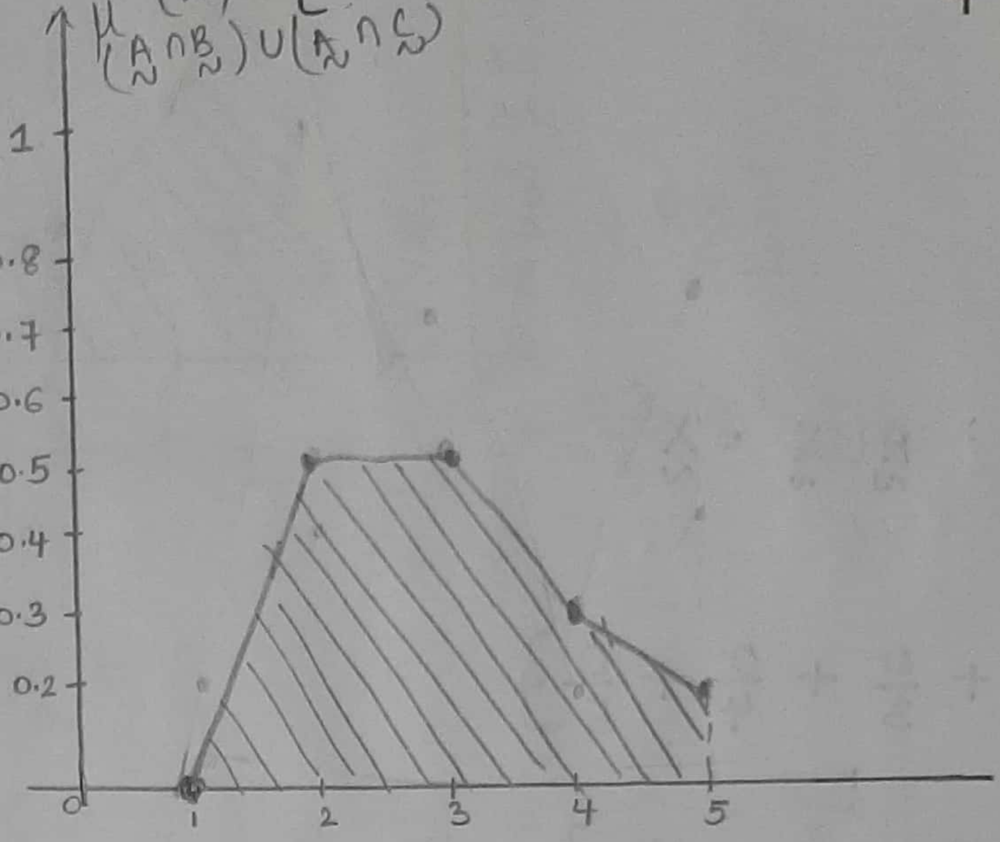
$$\underset{\sim}{B} \cup \underset{\sim}{C} = \left\{ \frac{0.2}{1} + \frac{0.5}{2} + \frac{0.7}{3} + \frac{0.3}{4} + \frac{0.7}{5} \right\}$$

$$\underset{\sim}{A} \cap (\underset{\sim}{B} \cup \underset{\sim}{C}) = \left\{ \frac{0}{1} + \frac{0.5}{2} + \frac{0.5}{3} + \frac{0.3}{4} + \frac{0.2}{5} \right\}$$

$$(\underset{\sim}{A} \cap \underset{\sim}{B}) = \left\{ \frac{0}{1} + \frac{0.5}{2} + \frac{0.5}{3} + \frac{0.2}{4} + \frac{0.2}{5} \right\}$$

$$(\underset{\sim}{A} \cap \underset{\sim}{C}) = \left\{ \frac{0}{1} + \frac{0.4}{2} + \frac{0.5}{3} + \frac{0.3}{4} + \frac{0.2}{5} \right\}$$

$$(\tilde{A} \cap \tilde{B}) \cup (\tilde{A} \cap \tilde{C})(x) = \left\{ \frac{0}{1} + \frac{0.5}{2} + \frac{0.5}{3} + \frac{0.3}{4} + \frac{0.}{} \right.$$



4. Identity

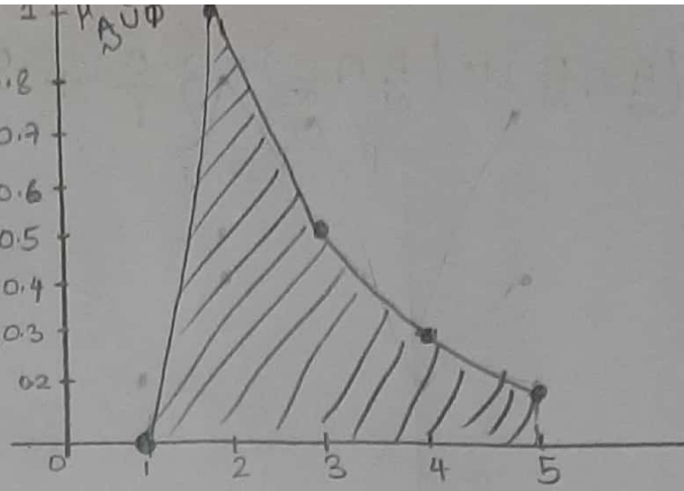
$$\tilde{A} \cup \phi = \tilde{A} \qquad \& \qquad \tilde{A} \cap X = \tilde{A}$$

$$\tilde{A} \quad \phi = \left\{ \frac{0}{1} + \frac{0}{2} + \frac{0}{3} + \frac{0}{4} + \frac{0}{5} \right\}$$

$$\tilde{A} \cup \phi = \left\{ \frac{0}{1} + \frac{1}{2} + \frac{0.5}{3} + \frac{0.3}{4} + \frac{0.2}{5} \right\}$$

$$\tilde{A} = \left\{ \frac{0}{1} + \frac{1}{2} + \frac{0.5}{3} + \frac{0.3}{4} + \frac{0.2}{5} \right\} \qquad \tilde{X} = \left\{ \frac{1}{1} \right.$$

$$\tilde{A} \cap \tilde{X} = \left\{ \frac{0}{1} + \frac{1}{2} + \frac{0.5}{3} + \frac{0.3}{4} + \frac{0.2}{5} \right\}$$



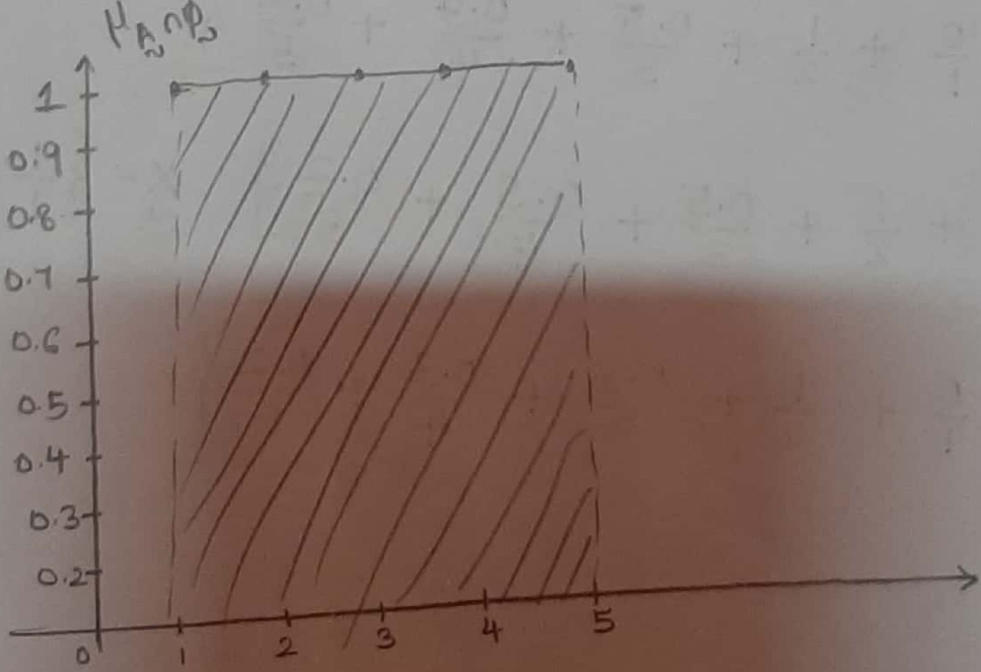
$\underset{\sim}{A} \cap \underset{\sim}{\phi} = \underset{\sim}{\phi}$ $\qquad$ $\underset{\sim}{A} \cup \underset{\sim}{X} = \underset{\sim}{X}$

$$\underset{\sim}{A} \cap \underset{\sim}{\phi} = \left\{ \frac{0}{1} + \frac{0}{2} + \frac{0}{3} + \frac{0}{4} + \frac{0}{5} \right\}$$

$$\underset{\sim}{\phi} = \left\{ \frac{0}{1} + \frac{0}{2} + \frac{0}{3} + \frac{0}{4} + \frac{0}{5} \right\}$$

$$\underset{\sim}{A} \cup \underset{\sim}{X} = \left\{ \frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} \right\}$$

$$\underset{\sim}{X} = \left\{ \frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} \right\}$$



5. Idempotency:

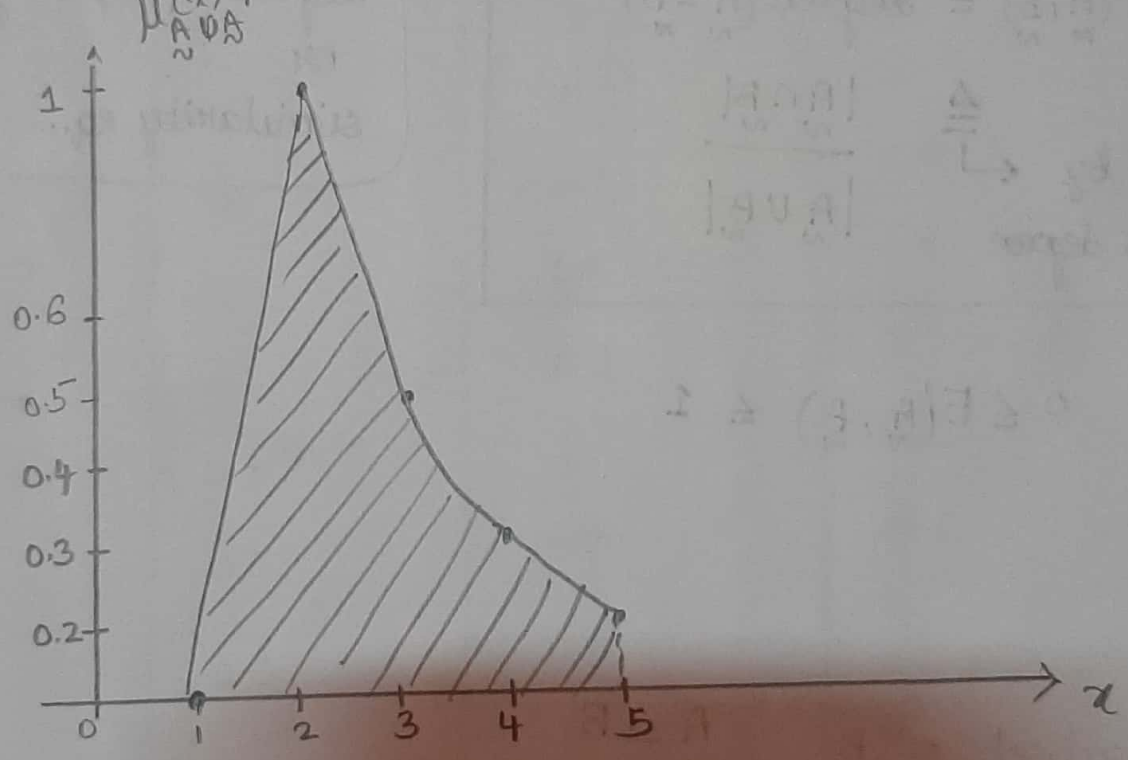
$$\underset{\sim}{A} \cup \underset{\sim}{A} = \underset{\sim}{A}$$

$$\underset{\sim}{A} = \left\{ \frac{0}{1} + \frac{1}{2} + \frac{0.5}{3} + \frac{0.3}{4} + \frac{0.2}{5} \right\}$$

$$\underset{\sim}{A} \cup \underset{\sim}{A} = \left\{ \frac{0}{1} + \frac{1}{2} + \frac{0.5}{3} + \frac{0.3}{4} + \frac{0.2}{5} \right\}$$

$$\underset{\sim}{A} \cap \underset{\sim}{A} = \underset{\sim}{A}$$

$$\underset{\sim}{A} \cap \underset{\sim}{A} = \left\{ \frac{0}{1} + \frac{1}{2} + \frac{0.5}{3} + \frac{0.3}{4} + \frac{0.2}{5} \right\}.$$



1. EQUALITY

membership equal of 2 sets → equal sets

Degree of equality - 0 to 1.

↳ completely equal

↳ no elements matching.

0.5

↳ half have same membership.

- Sets A & B are equal if & only if

iff $\boxed{\mu_{\tilde{A}}(x) = \mu_{\tilde{B}}(x)}$ $\forall$ $x \in X$ (or) U

belongs

such that

→ To check a degree of equality of 2 fuzzy sets, similarity measure is used

$$\boxed{E(\tilde{A}, \tilde{B}) \equiv \text{degree}(\tilde{A} = \tilde{B}) \triangleq \frac{|\tilde{A} \cap \tilde{B}|}{|\tilde{A} \cup \tilde{B}|}}$$

given by

represents degree

degree of equality

or

similarity eq.

$$0 \leq E(\tilde{A}, \tilde{B}) \leq 1$$

2. SUBSETS

A is a subset of B $A \subseteq B$

iff $\boxed{\mu_{\tilde{A}}(x) \leq \mu_{\tilde{B}}(x)}$, $\forall$ $x \in U$

- To check the degree that $A \subseteq B$, subset hood measure is used given by

$$S(\underset{\sim}{A}, \underset{\sim}{B}) \equiv \text{degree}(\underset{\sim}{A} \subseteq \underset{\sim}{B}) \triangleq \frac{|\underset{\sim}{A} \cap \underset{\sim}{B}|}{|\underset{\sim}{A}|}$$

Problem

Consider 2 membership funct of 2 fuzzy sets A & B given by $\underset{\sim}{A} = \{$

SCORE	HIGH SCORE ($\underset{\sim}{A}$)	Medium score ($\underset{\sim}{B}$)	Low score ($\underset{\sim}{C}$)
10	0	0	1
20	0	0	1
30	0	0.1	0.9
40	0	0.5	0.7
50	0.1	0.8	0.5
60	0.3	1	0.3
70	0.5	0.8	0.1
80	0.8	0.5	0
90	1	0	0
100	1	0	0

Calculate compliments $\overline{\underset{\sim}{A}}, \overline{\underset{\sim}{B}}, \overline{\underset{\sim}{C}}$, $\underset{\sim}{A} \cap \underset{\sim}{B}$, $\underset{\sim}{A} \cup \underset{\sim}{B}$, $E(\underset{\sim}{A}, \underset{\sim}{B})$, $S(\underset{\sim}{A}, \underset{\sim}{B})$

$$\underset{\sim}{A} = \left\{ \frac{0}{10} + \frac{0}{20} + \frac{0}{30} + \frac{0}{40} + \frac{0.1}{50} + \frac{0.3}{60} + \frac{0.5}{70} + \frac{0.8}{80} + \frac{1}{90} + \frac{1}{100} \right.$$

$$\underset{\sim}{B} = \left\{ \frac{0}{10} + \frac{0}{20} + \frac{0.1}{30} + \frac{0.5}{40} + \frac{0.8}{50} + \frac{1}{60} + \frac{0.8}{70} + \frac{0.5}{80} + \frac{0}{90} + \frac{0}{100} \right\}$$

$$\underset{\sim}{C} = \left\{ \frac{1}{10} + \frac{1}{20} + \frac{0.9}{30} + \frac{0.7}{40} + \frac{0.5}{50} + \frac{0.3}{60} + \frac{0.1}{70} + \frac{0}{80} + \frac{0}{90} + \frac{0}{100} \right\}$$

$0.1 + 0.3 + 0.5 + 0.5 = 1.4$

$$\underset{\sim}{A} \cap \underset{\sim}{B} = \left\{ \frac{0}{10} + \frac{0}{20} + \frac{0}{30} + \frac{0}{40} + \frac{0.1}{50} + \frac{0.3}{60} + \frac{0.5}{70} + \frac{0.8}{80} + \frac{0}{90} + \frac{0}{100} \right.$$

$$\underset{\sim}{A} \cup \underset{\sim}{B} = \left\{ \frac{0}{10} + \frac{0}{20} + \frac{0.1}{30} + \frac{0.5}{40} + \frac{0.8}{50} + \frac{1}{60} + \frac{0.8}{70} + \frac{0.8}{80} + \frac{1}{90} + \frac{1}{100} \right.$$

$0.1 + 0.5 + 0.8 + 1 + 0.8 + 0.8 + 1 + 1 = 6$

$\cancel{\overline{\underset{\sim}{A}} = \left\{ \frac{1}{10} + \right.}$

$$E(\underset{\sim}{A}, \underset{\sim}{B}) \triangleq \frac{|\underset{\sim}{A} \cap \underset{\sim}{B}|}{|\underset{\sim}{A} \cup \underset{\sim}{B}|}$$

$$\triangleq \frac{1.4}{6}$$

$$\triangleq 0.233$$

$$S(\underset{\sim}{A}, \underset{\sim}{B}) \triangleq \frac{|\underset{\sim}{A} \cap \underset{\sim}{B}|}{|\underset{\sim}{A}|}$$

$$\triangleq \frac{1.4}{3.7}$$

$$\triangleq 0.378$$

$$\overline{\underset{\sim}{A}} = \left\{ \frac{1}{10} + \frac{1}{20} + \frac{1}{30} + \frac{1}{40} + \frac{0.9}{50} + \frac{0.7}{60} + \frac{0.5}{70} + \frac{0.2}{80} + \frac{0}{90} + \frac{0}{100} \right\}$$

$$\overline{\underset{\sim}{B}} = \left\{ \frac{1}{10} + \frac{1}{20} + \frac{0.9}{30} + \frac{0.5}{40} + \frac{0.8}{50} + \frac{0}{60} + \frac{0.3}{70} + \frac{0.5}{80} + \frac{1}{90} + \frac{1}{100} \right\}$$

$$\overline{\underset{\sim}{C}} = \left\{ \frac{0}{10} + \frac{0}{20} + \frac{0.1}{30} + \frac{0.3}{40} + \frac{0.5}{50} + \frac{0.7}{60} + \frac{0.9}{70} + \frac{1}{80} + \frac{1}{90} + \frac{1}{100} \right\}.$$

Property:

4. law of contradiction:

$$\underset{\sim}{A} \cap \overline{\underset{\sim}{A}} \neq \phi$$

5. law of intermediate or law of excluded middle.

$$\underset{\sim}{A} \cup \overline{\underset{\sim}{A}} \neq X \text{ or } U$$

2. Excluded middle law $\quad \underset{\sim}{A} \cup \overline{\underset{\sim}{A}} \neq X$ or U

$$\underset{\sim}{A} \cap \overline{\underset{\sim}{A}} \neq \phi$$

2. Excluded middle law & law of contradiction

$$\underset{\sim}{A} \cup \overline{\underset{\sim}{A}} \neq X \text{ or } U$$

$$\underset{\sim}{A} \cap \overline{\underset{\sim}{A}} \neq \phi$$

ABSORPTION ~~LAW~~ : property

$$\underset{\sim}{A} \cup (\underset{\sim}{A} \cap \underset{\sim}{B}) = \underset{\sim}{A}$$

$$\underset{\sim}{A} \cap (\underset{\sim}{A} \cup \underset{\sim}{B}) = \underset{\sim}{A}$$

Law of zero :

$$\underset{\sim}{A} \cup X = X$$

$$\underset{\sim}{A} \cap \phi = \phi$$

Law of identity :

$$\underset{\sim}{A} \cup \phi = \underset{\sim}{A}$$

$$\underset{\sim}{A} \cap X = \underset{\sim}{A}$$

(or)

$$\underset{\sim}{A} \cup \phi = \underset{\sim}{A} \quad , \quad \underset{\sim}{A} \cap X = \underset{\sim}{A}$$

$$\underset{\sim}{A} \cup X = X \quad , \quad \underset{\sim}{A} \cap \phi = \phi$$

CARTESIAN PRODUCT :

universal no need ~

- Let $\underset{\sim}{A_1}, \underset{\sim}{A_2}, ----\underset{\sim}{A_n}$ be the fuzzy sets in $U_1, U_2, --- U_n$ respectively. The cartesian product of $\underset{\sim}{A_1}, \underset{\sim}{A_2}, --- \underset{\sim}{A_n}$ is a fuzzy set in the product space $U_1 \times U_2 \times --- \times U_n$ with with the membership funct as

$$\boxed{\mu_{A_1 \times A_2 \times ---- \times A_n}(x_1, x_2, ---- x_n) \triangleq \min\left[\mu_{\underset{\sim}{A_1}}(x_1), \mu_{\underset{\sim}{A_2}}(x_2), ------, \mu_{\underset{\sim}{A_n}}(x_n)\right]}$$

where $x_1 \in U_1$, $x_2 \in U_2$, --- $x_n \in U_n$.

Operations:

ALGEBRAIC SUM

The algebraic sum of 2 fuzzy sets $\underset{\sim}{A} + \underset{\sim}{B}$ is given by

$$\boxed{\mu_{\underset{\sim}{A}+\underset{\sim}{B}}(x) \triangleq \mu_{\underset{\sim}{A}}(x) + \mu_{\underset{\sim}{B}}(x) - \mu_{\underset{\sim}{A}}(x)\,\mu_{\underset{\sim}{B}}(x)}$$

. ALGEBRAIC PRODUCT

The algebraic product of 2 fuzzy set $\underset{\sim}{A} \cdot \underset{\sim}{B}$ is given by

$$\boxed{\mu_{\underset{\sim}{A}\cdot\underset{\sim}{B}}(x) \triangleq \mu_{\underset{\sim}{A}}(x) \cdot \mu_{\underset{\sim}{B}}(x)}$$

5. BOUNDED SUM

- Bounded sum of 2 fuzzy set $\underset{\sim}{A} \oplus \underset{\sim}{B}$ is given by

$$\boxed{\mu_{\underset{\sim}{A}\oplus\underset{\sim}{B}}(x) \triangleq \min\{1, \mu_{\underset{\sim}{A}}(x) + \mu_{\underset{\sim}{B}}(x)\}}$$

6. BOUNDED DIFFERENCE

- Bounded difference of 2 fuzzy sets $\underset{\sim}{A} \ominus \underset{\sim}{B}$ is given by

$$\boxed{\mu_{\underset{\sim}{A}\ominus\underset{\sim}{B}}(x) \triangleq \max\{0, \mu_{\underset{\sim}{A}}(x) - \mu_{\underset{\sim}{B}}(x)\}}$$

- let fuzzy set $\underset{\sim}{A} = \{(3, 0.5), (5, 1), (7, 0.6)\}$

$$\underset{\sim}{B} = \{(3, 1), (5, 0.6)\}$$

Calculate the cartesian product $\underset{\sim}{A} \times \underset{\sim}{B}$, algebraic sum $\underset{\sim}{A} + \underset{\sim}{B}$, algebraic prod $\underset{\sim}{A} \cdot \underset{\sim}{B}$, bounded sum $\underset{\sim}{A} \oplus \underset{\sim}{B}$, bounded diff $\underset{\sim}{A} \ominus \underset{\sim}{B}$.

max max min

Sol:- $\underset{\sim}{A} = \left\{ \frac{0.5}{3} + \frac{1}{5} + \frac{0.6}{7} \right\}$

$$\underset{\sim}{B} = \left\{ \frac{1}{3} + \frac{0.6}{5} \right\}$$

(i) Cartesian product $\triangleq \min\left[\mu_{\underset{\sim}{A_1}}(x_1), \mu_{\underset{\sim}{A_2}}\right.$

$$\underset{\sim}{A} \times \underset{\sim}{B} = \{[(3,3), 0.5], [(3,5), 0.5], [(5,3), 1], [(5,5), 0.6], [(7,3), 0.6], [(7,5), 0.6]\}$$

(ii) $\mu_{\underset{\sim}{A} + \underset{\sim}{B}}(x) \triangleq \mu_{\underset{\sim}{A}}(x) + \mu_{\underset{\sim}{B}}(x) - \mu_{\underset{\sim}{A}}(x) \cdot \mu_{\underset{\sim}{B}}(x)$

max (0.5 × 0.3 ×

$\frac{0.5}{3}$ $\frac{1}{3}$ ✓ $\frac{1}{5}$ $\frac{0.6}{5}$ $\frac{0}{7}$ $\frac{0.6}{7}$

$$\underset{\sim}{A} + \underset{\sim}{B} = \{(3, 1), (5, 1), (7, 0.6)\}$$

(iii) $\underset{\sim}{A} \cdot \underset{\sim}{B} = \{(3, 0.5), (5, 0.6), (7, 0)\}$

(iv) $\mu_{\underset{\sim}{A} \oplus \underset{\sim}{B}}(x) = \min\{1, \mu_{\underset{\sim}{A}}(x) + \mu_{\underset{\sim}{B}}(x)\}$

$\frac{0.5}{3}, \frac{1}{3}, \min(1, 0.5+1)$

$\underset{\sim}{A} \oplus \underset{\sim}{B} = \{(3, 1), (5, 1), (7, 0.6)\}$

$\min(1, 1+0.6) \quad \min(1, 0+0.6)$

(v) $\underset{\sim}{A} \ominus \underset{\sim}{B} = \max\{0, \mu_{\underset{\sim}{A}}(x) - \mu_{\underset{\sim}{B}}(x)\}$

$= \{(3, 0), (5, 0.4), (7, 0.6)\}$

EXTENSIONS OF FUZZY SETS CONCEPT.

* Ist direction

→ The first direction is, fuzzy sets including different structures of membership space & different assumption about membership function

∘ IInd direction

→ Other operations on fuzzy sets

I DIRECTION

other Kind of fuzzy sets

level 1 v/s level 2
Type 1 v/s Type 2

→ If fuzzy set whose membership funct is itself a fuzzy set is called Type-2 Fuzzy set

- Type 1 is ordinary fuzzy set. Type 2 fuzzy set is a fuzzy set whose membership value are type 1 fuzzy set on interval [0,1]

- A type-2 fuzzy sets in a universe 'U' is characterised by a fuzzy membership function $\mu_{\tilde{A}}$ as

$$\mu_{\underset{\sim}{A}} : U \rightarrow [0,1]^{[0,1]} \longrightarrow (1)$$

where $\mu_{\underset{\sim}{A}}(x)$ is a fuzzy grade and is a fuzzy set on the interval $[0,1]$ represented by

$$\mu_{\underset{\sim}{A}}(x) = \int \frac{f(u)}{u} , \quad u \in [0,1] \longrightarrow (2)$$

where the function f is a membership funct for fuzzy grade $\mu_{\underset{\sim}{A}}(x)$ & is defined as $f : [0,1] \rightarrow [0,1]$

- Example for type-2 :

We can define type-2 fuzzy set "beautiful" with membership value as type-1 fuzzy set such as below avg, avg, above avg, superior & so on.

→ Therefore we can define type-m fuzzy set $(m>1)$ in the universe 'U' whose membership values are type $(m-1)$ fuzzy set on the interval $[0,1]$

→ Different extension of the concept of fuzzy set is to consider a fuzzy set of fuzzy set of universe U i.e a fuzzy set whose elements are fuzzy set, Such fuzzy sets are called level-2 fuzzy sets.

* Type 2, level 2 are different.

Example for level-2 :

A level-2 fuzzy set is the collection of desired attribute for an electric razor. The elements of level 2 fuzzy sets are ordinary (level 1) fuzzy sets such as reliable, inexpensive, good appearance & so on.

→ A level-K fuzzy sets ($K>1$) can be defined where k indicates the depth of nesting.

- Given a universe 'U', let $\underset{\sim}{P}(U)$ denote the set of all fuzzy subset of 'U' & $\underset{\sim}{P}^K(U)$ is defined by

$$\underset{\sim}{P}^K(U) = \underset{\sim}{P}\left(P^{K-1}(U)\right)$$

For all integers $K \geq 2$ the level-K fuzzy set, $\underset{\sim}{A}$ is defined by

$\underset{\sim}{\mu_A} : \underset{\sim}{P}^{K-1}(U) \rightarrow [0,1]$

II DIRECTION

Further operations on fuzzy sets

($\cup$, $\cap$, compliment $\rightarrow$ non parametric operations)
type 1.

→ Different classes of functions for compliment, intersection & union are considered as ~~[illegible]~~ vowels

- These funct will possess different property, for each operation the corresponding funct can be divided into 2 categories.

category (1) Non parametric functions such as.
(ideal cases)

a) $\mu_{\bar{\underset{\sim}{A}}}(x) \triangleq 1 - \mu_{\underset{\sim}{A}}(x) \quad \forall\, x \in U$

b) $\mu_{\underset{\sim}{A} \cap \underset{\sim}{B}}(x) \triangleq \min\left[\mu_{\underset{\sim}{A}}(x), \mu_{\underset{\sim}{B}}(x)\right]$

$\equiv \mu_{\underset{\sim}{A}}(x) \wedge \mu_{\underset{\sim}{B}}(x)$

c) $\mu_{\underset{\sim}{A} \cup \underset{\sim}{B}}(x) \triangleq \max\left[\mu_{\underset{\sim}{A}}(x), \mu_{\underset{\sim}{B}}(x)\right]$

$\equiv \mu_{\underset{\sim}{A}}(x) \underset{\text{OR}}{\vee} \mu_{\underset{\sim}{B}}(x)$

Parametric functions in which parameters are used to adjust the strength of corresponding operation.

(i) COMPLIMENT : Fuzzy Complement

- $\bar{\underset{\sim}{A}}$ fuzzy complement $\bar{A}$ is specified by a function given by $c:[0,1] \longrightarrow [0,1]$ such that

$$\mu_{\bar{\underset{\sim}{A}}}(x) = c\left(\mu_{\underset{\sim}{A}}(x)\right)$$

parametric function

i.e $c(\cdot)$ ~~is monotonic non increasing~~ satisfy the following function

condition 1:

C1 : Boundary condition is given by $c(0)=1$

$c(1)=0$.

condition 2:

Monotonic property :

For any $x_1, x_2 \in U$ if

$\mu_{\underset{\sim}{A}}(x_1) < \mu_{\underset{\sim}{A}}(x_2)$ then

$$c(\mu_{\underset{\sim}{A}}(x_1)) \geq c(\mu_{\underset{\sim}{A}}(x_2))$$

It is a non increasing function
$c(\cdot)$

Condition 3 : Continuity

$c(\cdot)$ is a continous function.

Condition 4 : Involution:

→ $c(\cdot)$ is involutive i.e.

$$c(c(\mu_{\underset{\sim}{A}}(x))) = \mu_{\underset{\sim}{A}}(x) \quad \forall \; x \in U.$$

- Based on above conditions, typical examples of non parametric & parametric fuzzy compliment are shown below.

. Negation Compliment :

The compliment of $\underset{\sim}{A}$ using this operation is denoted by $\overline{\underset{\sim}{A}}$ & defined as

$$\mu_{\overline{\underset{\sim}{A}}}(x) = c(\mu_{\underset{\sim}{A}}(x)) \triangleq 1 - \mu_{\underset{\sim}{A}}(x) \quad \forall \; x \in U.$$

. λ Compliment (sugeno class) :

. This compliment is denoted by $\overline{\underset{\sim}{A}}^{\lambda}$ & is defined by

$$\mu_{\bar{\underset{\sim}{A}}^{\lambda}}(x) = c(\mu_{\underset{\sim}{A}}(x)) \triangleq \frac{1-\mu_{\underset{\sim}{A}}(x)}{1+\lambda\mu_{\underset{\sim}{A}}(x)}$$

$\lambda \rightarrow$ gives degree of compliment

$$-1 < \lambda < \infty$$

→ When $\lambda = 0$ the function becomes $c(\mu_{\underset{\sim}{A}}(x)) \triangleq 1-\mu_{\underset{\sim}{A}}(x)$

i.e the standard fuzzy compliment.

→ when $\lambda \rightarrow -1$; $\bar{\underset{\sim}{A}}^{\lambda} \longrightarrow U$

→ when $\lambda \rightarrow \infty$; $\bar{\underset{\sim}{A}}^{\lambda} \longrightarrow \emptyset$

3. W - COMPLEMENT (Yager class)

- The complement is denoted by $\bar{\underset{\sim}{A}}^{w}$

$$\mu_{\bar{\underset{\sim}{A}}^{w}}(x) = c(\mu_{\underset{\sim}{A}}(x))$$

$$\triangleq \left(1-\mu_{\underset{\sim}{A}}^{w}(x)\right)^{1/w}$$

$$0 < w < \infty$$

the parameter w adjust the degree of compliment, when $w=1$ the w compliment funct becomes std fuzzy complimen given by $c(\mu_{\underset{\sim}{A}}(x)) = 1-\mu_{\underset{\sim}{A}}(x)$

$w \to 0 \quad \bar{\underset{\sim}{A}}^{w} \to U$

$w \to \infty \quad \bar{\underset{\sim}{A}}^{w} \to \phi$

→ The equilibrium of fuzzy compliment c is defined as any value 'a' for which $c(a)=a$

$c(0.5) = 0.5$

Example :

The equilibrium of std complement operation is 0.5, which is the solution of equation $1-a=a$

- An imp property of all fuzzy comp is that every fuzzy complement has atmost one equilibrium due to monotonic non increasing nature of fuzzy compliment.

EXTENSION PRINCIPLE & ITS APPLICATION -

→ The extension principle allows the generalization of crisp mathematical concept to the fuzzy set frame work & extends point to point mapping to the mappings for fuzzy sets

- It provides a means for any function 'f' that maps n-tuple $(x_1, x_2, ----x_n)$ in crisp set 'U' to a point in crisp set 'V' to be generalized to mapping n fuzzy sub sets in 'U' to a fuzzy sub set in 'V'.

- Any mathematical relationship b/w non fuzzy element can be extended to deal with fuzzy entities

- The extension principle is very useful in dealing with set-theoretic operation for higher order fuzzy set.

- Given a function $f: U \rightarrow V$ is a fuzzy set 'A' in universe 'U' where

$$\underset{\sim}{A} = \frac{\mu_1}{x_1} + \frac{\mu_2}{x_2} + --- + \frac{\mu_n}{x_n}$$

- The extension principle states that

$$f(\underset{\sim}{A}) = f\left(\frac{\mu_1}{x_1} + \frac{\mu_2}{x_2} + \cdots + \frac{\mu_n}{x_n}\right)$$

$$= \frac{\mu_1}{f(x_1)} + \frac{\mu_2}{f(x_2)} + \cdots + \frac{\mu_n}{f(x_n)}$$

→ If more than one element of 'U' is mapped to the same element 'y' in 'V' by the function 'f' (many-to-one mapping) (max wala lena) then max amt their membership grade is taken, express by

$$\mu_{f(\underset{\sim}{A})}(y) = \max_{\substack{x_i \in U \\ f(x_i)=y}} \left[\mu_{\underset{\sim}{A}}(x_i)\right]$$

where x_i are the elements that are mapped to same element y

- The funct 'f' maps n-tuples in U to a point in 'V'

- Let 'U' be the cartesian product of universes

$U = U_1 \times U_2 \times \cdots \times U_n$ & $\underset{\sim}{A_1}, \underset{\sim}{A_2}, \cdots \underset{\sim}{A_n}$ be n fuzzy sets in $U_1, U_2, \cdots U_n$ respectively.

The funct f maps n-tuples $(x_1, x_2, \cdots x_n)$ to crisp set

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to a point 'y' in crisp set 'V' i.e

$$y = f(x_1, x_2, ---, x_n).$$

The extension principle allows the function

$f(x_1, x_2, -- x_n)$ to be extended to act on 'n' fuzzy

subset of 'U', $A_1, A_2, -- A_n$ such that

$B = f(A)$ where B is the fuzzy image (fuzzy set)

of $A_1, A_2, -- A_n$. through the function $f(\cdot)$

- The fuzzy set B is defined by

$$\underset{\sim}{B} = \left\{ (y, \mu_{\underset{\sim}{B}}(y)) \,\middle|\, \overset{\text{for}}{y} = f(x_1, x_2, ---x_n), \; (x_1, x_2, ---, x_n) \in U \right\}$$

where $\mu_{\underset{\sim}{B}}(y)$ is given by sup (support).

$$\mu_{\underset{\sim}{B}}(y) = \sup_{\substack{(x_1, x_2, --, x_n) \in U \\ y = f(x_1, x_2, -- x_n)}} \min\left[\mu_{\underset{\sim}{A_1}}(x_1), \mu_{\underset{\sim}{A_2}}(x_2) ---- \mu_{\underset{\sim}{A_n}}(x_n)\right]$$

with additional condition

$\mu_{\underset{\sim}{B}}(y) = 0$ if there exist no $(x_1, x_2, \ldots x_n) \in U$ s

that $y = f(x_1, x_2, \ldots x_n)$

PROBLEM:

→ Let $U = \{1,2,3,4,5,6,7,8,9,10\}$ a fuzzy set $\underset{\sim}{A}$ = " Large

given as $\underset{\sim}{A}$ = "Large" $= \left\{ \frac{0.5}{6} + \frac{0.7}{7} + \frac{0.9}{9} + \frac{1}{10} \right\}$

The function f is a squaring operation given by

$y = f(x) = x^2$. Apply extension principle.

Sol: B = "Large^2" $= \left\{ \frac{\mu_1}{f(x_1)} + \frac{\mu_2}{f(x_2)} + \cdots + \frac{\mu_n}{f(x_n)} \right\}$

$$= \left\{ \frac{0.5}{36} + \frac{0.7}{49} + \frac{0.9}{81} + \frac{1}{100} \right\}$$

- The extension principle can be used to define operation of interaction (algebraic product), union (algebric sum), compliment of type-2 fuzzy set.

- Let $\mu_{\tilde{A}}(x)$ & $\mu_{\tilde{B}}(x)$ be fuzzy grades for type-2 fuzzy sets A & B resp.

- They can be defined as

$$\mu_{\tilde{A}}(x) = \int \frac{f(u)}{u} \; ; \quad u \in [0,1]$$

↳ not integration, only notation

$$\mu_{\tilde{B}}(x) = \int \frac{g(w)}{w} \; ; \quad w \in [0,1]$$

where f & g depends on x as well as u or w

Using the extension principle we have different operation on type-2 fuzzy sets as follows.

1. MIN OPERATOR $[\underset{\sim}{A} \cap \underset{\sim}{B}]$:

$$\mu_{\underset{\sim}{A} \cap \underset{\sim}{B}}(x) = \mu_{\underset{\sim}{A}}(x) \cap \mu_{\underset{\sim}{B}}(x) = \int \frac{f(u)}{u} \cap \int \frac{g(w)}{w}$$

$$= \int \frac{f(u) \wedge g(w)}{u \wedge w}$$

$\wedge \rightarrow$ and

$\vee \rightarrow$ or.

2. MAX OPERATOR $[\underset{\sim}{A} \cup \underset{\sim}{B}]$:

$$\mu_{\underset{\sim}{A} \cup \underset{\sim}{B}}(x) = \mu_{\underset{\sim}{A}}(x) \cup \mu_{\underset{\sim}{B}}(x) = \int \frac{f(u)}{u} \cup \int \frac{g(w)}{w}$$

$$= \int \frac{f(u) \wedge g(w)}{u \vee w}$$

3. ALGEBRAIC PRODUCT $[\underset{\sim}{A} \cdot \underset{\sim}{B}]$:

$$\mu_{\underset{\sim}{A} \cdot \underset{\sim}{B}}(x) = \mu_{\underset{\sim}{A}}(x) \cdot \mu_{\underset{\sim}{B}}(x) = \int \frac{f(u)}{u} \cdot \frac{g(w)}{w}$$

$$= \int \frac{f(u) \wedge g(w)}{u \cdot w}$$

4. ALGEBRAIC SUM $[\underset{\sim}{A} \hat{+} \underset{\sim}{B}]$:

$$\mu_{\underset{\sim}{A} \hat{+} \underset{\sim}{B}}(x) = \mu_{\underset{\sim}{A}}(x) \hat{+} \mu_{\underset{\sim}{B}}(x) = \int \frac{f(u) \wedge g(w)}{u \hat{+} w}$$

$$= \int \frac{f(u) \wedge g(w)}{u + w - uw}.$$

COMPLEMENT $[\bar{\underset{\sim}{A}}]$:

$$\mu_{\bar{\underset{\sim}{A}}}(x) = \overline{\mu_{\underset{\sim}{A}}(x)} = \int \frac{f(u)}{(1-u)}$$

PROBLEM :

Let $\mu_{\underset{\sim}{A}}(x) = \frac{0.3}{0.4} + \frac{0.7}{0.8}$ & $\mu_{\underset{\sim}{B}}(x) = \frac{0.1}{0.1} + \frac{0.5}{0.2} + \frac{1}{0.4}$

then calculate - algebraic product
- algebraic sum
- $\underset{\sim}{A} \cap \underset{\sim}{B}$
- $\underset{\sim}{A} \cup \underset{\sim}{B}$
- $\bar{\underset{\sim}{A}}$
- $\bar{\underset{\sim}{B}}$

$$\mu_{\underset{\sim}{A} \hat{+} \underset{\sim}{B}}(x) = \mu_{\underset{\sim}{A}}(x) \hat{+} \mu_{\underset{\sim}{B}}(x) = \int \frac{f(u) \wedge g(w)}{u \hat{+} w}$$

$$= \int \frac{f(u) \wedge g(w)}{u + w - uw}.$$

5. COMPLEMENT $[\bar{\underset{\sim}{A}}]$:

$$\mu_{\bar{\underset{\sim}{A}}}(x) = \overline{\mu_{\underset{\sim}{A}}(x)} = \int \frac{f(u)}{(1-u)}$$

PROBLEM :

→ Let $\mu_{\underset{\sim}{A}}(x) = \frac{0.3}{0.4} + \frac{0.7}{0.8}$ & $\mu_{\underset{\sim}{B}}(x) = \frac{0.1}{0.1} + \frac{0.5}{0.2} + \frac{1}{0.4}$

then calculate - algebraic product

- algebraic sum

- $\underset{\sim}{A} \cap \underset{\sim}{B}$

- $\underset{\sim}{A} \cup \underset{\sim}{B}$

- $\bar{\underset{\sim}{A}}$

- $\bar{\underset{\sim}{B}}$

Sol: $\mu_{\underset{\sim}{A}}(x) = \frac{0.3}{0.4} + \frac{0.7}{0.8}$

$\mu_{\underset{\sim}{B}}(x) = \frac{0.1}{0.1} + \frac{0.5}{0.2} + \frac{1}{0.4}$

(i) $\mu_{\underset{\sim}{A}\cdot\underset{\sim}{B}}(x) = \mu_{\underset{\sim}{A}} \cdot \mu_{\underset{\sim}{B}} = \int \frac{f(u) \wedge g(w)}{u \cdot w}$

$= \frac{0.3 \wedge 0.1}{0.4 \times 0.1} + \frac{0.3 \wedge 0.5}{0.4 \times 0.2} + \frac{0.3 \wedge 1}{0.4 \times 0.4} + \frac{0.7 \wedge 0.1}{0.8 \times 0.1}$

$+ \frac{0.7 \wedge 0.5}{0.8 \times 0.2} + \frac{0.7 \wedge 1}{0.8 \times 0.4}$

max max

$\mu_{\underset{\sim}{A}\cdot\underset{\sim}{B}}(x) = \frac{0.1}{0.04} + \frac{0.3}{0.08} + \frac{0.3}{0.16} + \frac{0.1}{0.08} + \frac{0.5}{0.16} + \frac{0.7}{0.32}$

$= \frac{0.1}{0.04} + \frac{0.3}{0.08} + \frac{0.5}{0.16} + \frac{0.7}{0.32}$

(ii) $\mu_{\underset{\sim}{A} \hat{+} \underset{\sim}{B}}(x) = \int \frac{f(u) \wedge g(w)}{u + w - uw}$

$= \frac{0.3 \wedge 0.1}{0.4 + 0.1 - (0.4 \times 0.1)} + \frac{0.3}{0.4 + 0.2 - 0.4 \times 0.2} + \frac{0.3}{0.4 + 0.4 - 0.4 \times 0.4}$

$+ \frac{0.1}{0.8 + 0.1 - 0.8 \times 0.1} + \frac{0.5}{0.8 + 0.2 - 0.8 \times 0.2} + \frac{0.7}{0.8 + 0.4 - 0.8 \times 0.4}$

$= \frac{0.1}{0.46} + \frac{0.3}{0.52} + \frac{0.3}{0.64} + \frac{0.1}{0.82} + \frac{0.5}{0.84} + \frac{0.7}{0.88}$

(iii) $\mu_{\underset{\sim}{A} \cap \underset{\sim}{B}} = \int \frac{f(u) \wedge g(w)}{u \wedge w}$

$\wedge \rightarrow$ and min

$\vee \rightarrow$ or max

$$\mu_{\underset{\sim}{A}}(x) = \frac{0.3}{0.4} + \frac{0.7}{0.8}$$

$$\mu_{\underset{\sim}{B}}(x) = \frac{0.1}{0.1} + \frac{0.5}{0.2} + \frac{1}{0.4}$$

$$\mu_{\underset{\sim}{A} \cap \underset{\sim}{B}} = \frac{0.1}{0.1} + \frac{0.3}{0.2} + \frac{0.3}{0.4} + \frac{0.1}{0.1} + \frac{0.5}{0.2} + \frac{0.7}{0.4}$$

v) $\mu_{\underset{\sim}{A} \cup \underset{\sim}{B}} = \int \frac{f(u) \wedge g(w)}{u \vee w}$

$$= \frac{0.1}{0.4} + \frac{0.3}{0.4} + \frac{0.3}{0.4} + \frac{0.1}{0.8} + \frac{0.5}{0.8} + \frac{0.7}{0.8}$$

i) $\mu_{\bar{\underset{\sim}{A}}} = \frac{0.3}{0.6} + \frac{0.7}{0.2}$ $= \int \frac{f(u)}{1-u}$

$$\mu_{\bar{\underset{\sim}{B}}} = \frac{0.1}{0.9} + \frac{0.5}{0.8} + \frac{1}{0.6}.$$

CONSISTENCY DEGREE OF TWO FUZZY SETS:

- The consistency degree of two fuzzy sets $\underset{\sim}{A}$ & $\underset{\sim}{B}$ after applying extension principle to the equality of 2 fuzzy sets can be defined as

$$\underset{x=y}{\overset{\text{support}}{\sup}} \min\left(\mu_{\underset{\sim}{A}}(x), \mu_{\underset{\sim}{B}}(x)\right) = \text{Height}\left(\underset{\sim}{A} \cap \underset{\sim}{B}\right)$$

- Let the fuzzy set $\underset{\sim}{A}(\underset{\sim}{B})$ denote the degree of membership of fuzzy set $\underset{\sim}{B}$ in fuzzy set $\underset{\sim}{A}$. Using extension principle we have

$$\mu_{\underset{\sim}{A}(\underset{\sim}{B})}(i) = \sup_{\substack{x \in U \\ \mu_{\underset{\sim}{A}}(x) = i}} \left(\mu_{\underset{\sim}{B}}(x)\right) \quad \overset{\text{for all}}{\forall} \; i \in [0,1]$$